Class Notes (Test 2)

Chapter 2:
Multiplying and Dividing fractions

2.1 *Fractions are one way to write parts of a whole.
The top # is the numerator \( \Rightarrow \) (the part of the whole)
The bottom # is the denominator \( \Rightarrow \) (how many equal parts)

(ex.) \( \frac{5}{8} \)

5 parts shaded
8 equal parts of the whole

2 types of Fractions:
* Proper fraction
(a # less than 1)
Numerator is smaller than denominator

ex.) \( \frac{3}{4}, \frac{4}{7}, \frac{1}{9}, \frac{2}{15} \)

* Improper fraction
(a # greater than 1)
Numerator is the same as or larger than the denominator

ex.) \( \frac{7}{4}, \frac{5}{3}, \frac{9}{2}, \frac{4}{1} \)

2.2 Mixed Numbers
Whole numbers and fractions written together are called “mixed numbers.”
ex.) \( 5\frac{1}{4}, 6\frac{7}{8} \)

* To convert mixed #’s to improper fractions:
\( 5\frac{1}{4} \Rightarrow 4\times5+1 = 21 \Rightarrow \frac{21}{4} \) is the result.
\( 6\frac{7}{8} \Rightarrow 8\times6+7 = 55 \Rightarrow \frac{55}{8} \) is the result.
Note: the denominator stays the same.
* to convert improper fractions to mixed #\'s:

\[
\frac{17}{5} \Rightarrow \frac{3}{17} \quad \text{whole #} \quad 3 \frac{2}{5}
\]

2.3

Factors

Factors are numbers multiplied together to get a product.

ex) factors of 12: 1 x 12, 2 x 6, 3 x 4

\[
(1, 2, 3, 4, 6, 12)
\]

factors of 30: 1 x 30, 2 x 15, 3 x 10, 5 x 6

\[
(1, 2, 3, 5, 6, 10, 15, 30)
\]

Composite / Prime #\'s

* More than 1 factor
* Only 1 pair (itself and 1)

Prime factorizations (I prefer the division method)

ex)

\[
\begin{array}{c|cc}
2 & 48 \\
2 & 24 \\
2 & 12 \\
3 & 6 \\
3 & 3 \\
\hline
& 1
\end{array}
\]

\[
2 \times 2 \times 2 \times 2 \times 3
\]

Note: The exponent tells how many times the "2" was used.
2.4 Writing Fractions in lowest terms

A fraction is in lowest terms when the numerator and denominator have no common factor other than "1".

* All fraction answers must be in lowest terms.

Preferred methods:
1) dividing both #s by the GCF.
2) repeated division of both #'s until no common factor is left

ex) \( \frac{24}{42} = \frac{4}{7} \) or \( \frac{24}{42} = \frac{12}{21} = \frac{4}{7} \)

\( \frac{72}{18} = \frac{4}{7} \)

\( \frac{72}{18} = \frac{36}{9} \frac{2}{1} = \frac{4}{7} \)

Equivalent fractions

If two fractions are equivalent, their cross products will be equal.

ex) \( \frac{20}{4} \) and \( \frac{110}{22} \) \( \Rightarrow \frac{20}{4} \times \frac{22}{110} \)

\( 440 = 440 \)

2.5 Multiplying fractions

Multiply across \( \frac{\text{numerator} \times \text{numerator}}{\text{denominator} \times \text{denominator}} \)

ex) \( \frac{5}{8} \times \frac{3}{4} \Rightarrow \frac{5 \times 3}{8 \times 4} = \frac{15}{32} \)

Cancelling should be used if there are common factors between the top #’s & bottom #’s.

Cancelling is reducing to lowest terms before you multiply.
ex. \[
\left( \frac{6}{11} \div \frac{8}{8} \right) \quad (6 + 8 \text{ have a common factor of 2})
\]
\[
\Rightarrow \frac{3}{11} \div \frac{7}{84}
\]
\[
= \frac{21}{44} \quad \text{lowest terms}
\]
\[
\frac{2}{15} \times \frac{8}{15} = \frac{4}{25}
\]
\[
\frac{3}{5} \times \frac{5}{1} = \frac{27}{5}
\]

Area of a Rectangle

\[
\text{Area} = \text{Length} \times \text{Width}
\]

* units should be "squared"

ex) ft², yd², mi² or sq. ft, sq. yd, sq. mi

2.6 Applications of Multiplication

* note indicator words on page 147.

A fraction next to the word "of" indicates to multiply.

ex) \( \frac{2}{3} \text{ of 2500 children} \)

\[
\frac{2}{3} \times \frac{2500}{1} = 1000 \text{ children}
\]

2.7 Dividing Fractions

The "reciprocal" of a fraction is found by flipping the fraction over. Ex) \( \frac{10}{17} \Rightarrow \frac{17}{10} \)

To divide fractions, flip the 2nd (or bottom) fraction over and change to multiplication.
ex.) \[
\frac{7}{8} \div \frac{15}{16} = \frac{7}{8} \times \frac{16}{15} = \frac{7 \times 16}{8 \times 15} = \frac{14}{15}
\]

\[
\frac{3}{4} \div \frac{2}{5} = \frac{3}{4} \times \frac{5}{2} = \frac{15}{8}
\]

\[
\frac{4}{5} \div 4 = \frac{4}{5} \times \frac{1}{4} = \frac{4 \times 1}{5 \times 4} = \frac{2}{15}
\]

* Helpful hints when doing word problems involving multiplication/division.

<table>
<thead>
<tr>
<th>Note: Indicator words for division page 158</th>
</tr>
</thead>
<tbody>
<tr>
<td>1) When the units are the same ( \div )</td>
</tr>
<tr>
<td>2) When the units are different ( \times )</td>
</tr>
<tr>
<td>3) A fraction next to the word &quot;of&quot; ( \times )</td>
</tr>
</tbody>
</table>

ex) 1) How many \( \frac{2}{3} \)-gallon garden sprayers can be filled from 36 gallons of insect spray?

Both units are "gallons" so use division

\[
36 \div \frac{2}{3} = 36 \times \frac{3}{2} = \frac{108}{2} = 54 \text{ sprayers}
\]

2) A new vaccine is synthesized at the rate of 2 grams per day. How many grams can be synthesized in 12 days?

The units "grams" & "days" are different so use multiplication.

\[
2 \times 12 = 24
\]
2.8 **Multiplying and Dividing mixed numbers**

1) Change each mixed number to an improper fraction.
2) Cancel if you can (flip first if you are dividing!)
3) Simplify by multiplication

**Example:**

\[ 3 \frac{1}{2} \times 6 \frac{1}{3} = 3 \frac{11}{6} = 5 \frac{5}{6} \]

\[ 9 \frac{1}{4} \times 6 \frac{1}{2} = 9 \frac{3}{8} \times 3 = 27 \frac{3}{8} \]

\[ 1 \frac{1}{8} \div 2 \frac{1}{4} = \frac{9}{8} \div \frac{9}{4} = \frac{1}{2} \]

\[ 2 \frac{3}{4} \div 2 = \frac{11}{4} \div \frac{1}{2} = \frac{11}{2} = 5 \frac{1}{2} \]
Estimating mixed numbers

When working with mixed numbers, it is a good idea to estimate the answer first to make sure the final answer is reasonable.

To estimate the answer, round each mixed number to the nearest whole number.

Rounding Rules:
* If the numerator is half of the denominator or more, round the whole number part up.
* If the numerator is less than half the denominator, leave the whole number part the same.

Examples:

\[ 3 \frac{1}{2} \cdot 6 \frac{1}{3} \]

Estimate: \(4 \cdot 6 = 24\)

Exact: \(\frac{7}{2} \cdot \frac{19}{3} = \frac{133}{6} = 22 \frac{1}{6}\)

\[ 3 \frac{1}{2} \rightarrow 3 \text{ rounds to 4 because "1" is half of "2"} \]

\[ 6 \frac{1}{3} \rightarrow 6 \text{ stays the same because "1" is less than half of "3"} \]

\[ 5 \frac{2}{3} \div 6 \]

Estimate: \(6 \div 6 = 1\)

Exact: \(\frac{17}{3} \div \frac{1}{6} = \frac{17}{3} \cdot \frac{6}{1} = \frac{17}{1} = \frac{17}{1} = \frac{17}{1}\)

\[ 5 \frac{2}{3} \rightarrow 5 \text{ rounds to 6 because "2" is more than half of "3"} \]

\[ 6 \rightarrow 6 \text{ stays the same because it doesn't have a fraction to round it up.} \]